

Oriented manifold-based error tracking control for nonholonomic wheeled autonomous vehicles on Lie group for curvilinear approach

Control de rastreo de error en una variedad orientada para vehículos no holonómicos sobre un grupo de Lie en el enfoque curvilíneo

DOI: 10.46814/lajdv3n6-011

Recebimento dos originais: 01/10/2021

Aceitação para publicação: 23/11/2021

C. A. Peña Fernández

Department of Electrical Engineering Federal Institute of Bahia, Salvador - Brazil

E-mail: cesarfernandez@ifba.edu.br/ cefer86@gmail.com

ABSTRACT

This paper considers the trajectory tracking control of wheeled autonomous vehicles (WAV) with slipping in the wheels, i.e., when the kinematic constraints are not satisfied. Usually, the coordinates system used to represent all control problems suggest invariant subspaces mutually orthogonal, but this approach can not be enough to treat curvatures significative large at different navigation speed. In order to get a slight improvement on this topic, there are previous works showing that the kinematic problem (commonly associated with an outer loop) can be resynthesized by using other invariant subspaces, i.e., another representation of the configuration space. For this reason, the proposal reported here uses an oriented-manifold parametrized by a coordinate system on a curve viewpoint of the trajectory to describe the kinematic problem, however, the dynamic control law remains faithful to the singular perturbation approach with invariant subspaces mutually orthogonal, thus, it is possible to include the flexibility through a small factor in the dynamic model (well-known as ϵ), responsible to avoid the good-performance of the kinematic constraints. Only a common curvature-transformation between orthogonal and curve coordinates will be used to couple both approaches. Finally, it will be observed that when the controller is applied to the control scheme the behavior of the tracking is meaningfully improved.

Keywords: Error tracking control, flexibility, oriented-manifold, singular perturbations, nonlinear control.

RESUMEN

Este trabajo considera el control de seguimiento de trayectorias de vehículos autónomos con ruedas (WAV) con deslizamiento en las mismas, es decir, cuando las restricciones cinemáticas no se satisfacen. Normalmente, el sistema de coordenadas utilizado para representar todos los problemas de control sugiere subespacios invariantes mutuamente ortogonales, pero este enfoque puede no ser suficiente para tratar curvaturas significativamente grandes a diferentes velocidades de navegación. Para conseguir una ligera mejora en este tema, existen trabajos previos que muestran que el problema cinemático (comúnmente asociado a un bucle exterior) puede ser resintetizado utilizando otros subespacios invariantes, es decir, otra representación del espacio de configuración. Por esta razón, la propuesta reportada aquí utiliza un manifold orientado parametrizado por un sistema de coordenadas en un punto de vista de la curva de la trayectoria para describir el problema cinemático, sin embargo, la ley de control dinámico sigue siendo fiel al enfoque de perturbación singular con subespacios

invariantes mutuamente ortogonales, por lo tanto, es posible incluir la flexibilidad a través de un pequeño factor en el modelo dinámico (conocido como ε), responsable de evitar el buen funcionamiento de las restricciones cinemáticas. Sólo se utilizará una curvatura-transformación común entre coordenadas ortogonales y curvas para acoplar ambos enfoques. Finalmente, se observará que cuando se aplica el controlador al esquema de control el comportamiento del seguimiento mejora significativamente.

Palabras clave: Control de seguimiento de errores, flexibilidad, manifiesto orientado, perturbaciones singulares, control no lineal.

1 INTRODUCTION

Control of wheeled autonomous vehicles (WAV) has been a challenge in last years, mainly when the tracking trajectory is not suitable according to specifications of the workspace, taking into account that such specifications can be modeled by using the configuration space (due to different kinds of wheels and inherent contact constraints) or by considering nonlinear forces, either due to static friction nature on the wheels or by just torsional or flexible behavior on mechanical-coupling, all WAVs are always a new opportunity to research more alternatives to control them [1, 2, 3, 4, 5, 6, 7]. Usually, when WAVs do not satisfy kinematics constraints are treated as systems with a high non-holonomy degree, becoming more complex to synthesize any nonlinear control law that operates on the actuator's space and guarantees good convergence of the wheel speed as well as of the error for trajectory tracking. Hence, the growing interest to synthesize feedback control laws for mechanical systems subject to nonholonomic constraints characterizes a powerful research line, due to its challenging theoretical nature where does not exist a smooth pure state feedback control law¹ to guarantee the error converges to the origin [9, 10, 11, 12, 6]. And really, it can not fail to mention that the problem of finding smooth pure state feedback is rather impracticable because for one side the feedback control error requires some data before the wheels begin to move, even for situations with holonomy degree, as with mobile robots based on Swedish wheels [6, 7, 13, 14, 15]. From a geometrical viewpoint finding a nonlinear control law that improves the performance of the tracking error can be understood as finding an invariant m -manifold, being m the dimension of the domain associated with the variables that represent the violation of the kinematic constraints. This approach usually is well-known as slow manifold and has its beginning in the theory of singular perturbations [16, 17, 7, 18, 19, 9]. Nevertheless, this approach becomes rather impracticable when the coordinates system is based on an orthogonal basis (e.g., a cartesian coordinates system) since in significantly large curvatures for high speed these kinds of coordinates do not allow consistent enough data to fix deviations at the right time before to happen the curve. In summary, these are some reasons to apply

more theoretical frameworks based on manifolds composed of differentiable operations, also known as Lie groups.

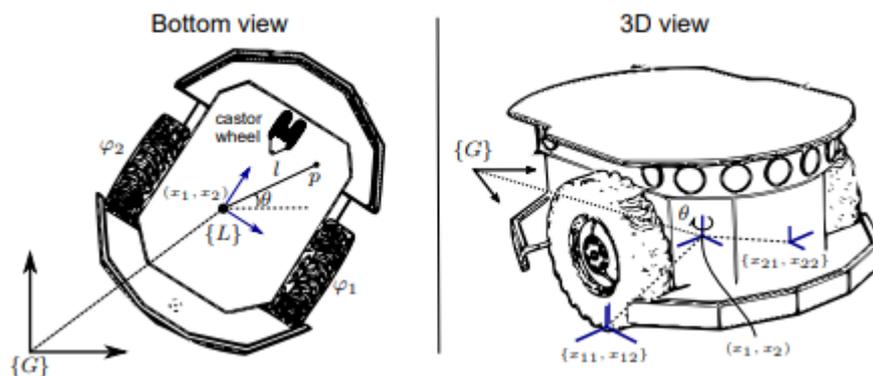
Manifolds are widely used to treat problems with WAV that do not satisfy the kinematic constraints and to this end, several approaches employ implicitly a manifold to linearize or synthesize a control law nevertheless such tradition is not enough when specific conditions of the rolling are omitted like happens with conditions of pure rolling and non-slipping, which are not satisfied properly at each time instant [7, 20, 21, 7]. It is the case of the backstepping technique where controllers are based on assumption that the full set of kinematic constraints is satisfied but the performance of the controllers does not offer a suitable adaptive response in front of some variations of the navigation speed [22, 23]. Past works developed by the author have shown nonlinear controllers with slightly increased performance when the weighted flexibility and the data of the constraint violation was included in an outer loop based on feedforward law (in order to avoid a pure feedback principle) [6], at the same way it was synthesized an inner loop associated with an invariant slow manifold and Poincar-Lindstedt method on cyclic phase diagrams [6, 7]. In contrast, in order to mitigate this difficulty, several types of controllers have been proposed, such as time-varying control laws, discontinuous control laws, and their respective hybrid control versions [5, 24]. Nevertheless, the techniques for trajectory control have been based on linearization techniques for local controlling [25, 26, 27, 28] or techniques of nonlinear state feedback with singular parameters [29, 20, 17] without taking into account the error propagation on closed-loop control scheme due to the flexibility. To this end, previous works of the author have shown the relevant role of the flexibility at each stage of the control scheme, it is the case of the controller based on the DCSV approach² to find suitable tuning of the parameters of an auxiliary control law on the outer loop according to the flexibility parameter ε [6, 31, 32, 30]. However, this approach imposes that the flexibility rate is null.

In this paper, we consider the tracking control problem of WAVs which are subjected to slipping effects, i.e., when the nonholonomic kinematic constraint of pure rolling is transgressed during the motion. In principle, this fact, which is related to deformability or flexibility of the wheels has already been treated at previous cited approaches however such ways suggest fixed flexibility on time for any condition of the surface, assuming that the wheels and surface remain equal to initial conditions. For this reason, the research reported here includes the flexibility parameter through a non-negative function into the kinematic controller. In addition, it will be used a suitable curvilinear approach to synthesize the kinematic control law and avoid data inconsistency when the speed is increased. Complementary, the manifold approach, as well as the use of a slow manifold analysis to synthesize a suitable control scheme (kinematic and dynamic problem), have been used according to

the previous studies (see [30, 31, 32, 6, 7]) and an oriented manifold whose differential 1-forms can be associated with the tracking error and the control actions of the kinematic controller such that it will guarantee the basic properties of a Lie group. On this structure, and with support of the singular perturbations theory and the models widely used by the author in previous works (see [30, 31, 32, 6, 7]) a feedback control law that uses data from kinematic controller will be synthesized.

This paper is organized as follows: In Section 2 is showed the mathematical model at previous works of the author and the preliminaries foundations associated with the curvilinear approach, necessary to find a kinematic control law. In Section 3 is used a strong concept of the oriented manifold (supported on the well-known Stokes theorem) and its application to build two 1-forms, the trajectory tracking error and the control action. In Section 4 a simulation is done by using parameters of the well-known WAV Pioneer 3dX through software MATLAB and V-REP. Finally, conclusions and final remarks are made in Section 5.

Figure 1: WAV with two fixed standard wheels, $\{x_1, x_2, \theta\}$ describe the cartesian position and guidance of the body frame $\{L\}$ into the global and fixed frame $\{G\}$ while the set $\{\phi_1, \phi_2\}$ describe the angular position of each wheel. The wheels associated with angles ϕ_1, ϕ_2 are standard fixed wheels with three DOF, only rotation around the normal vector of the wheel plane and translation without steering on their individual frames, located at poses $\{x_{11}, x_{12}\}$ and $\{x_{21}, x_{22}\}$ with reference to the fixed global frame. The castor wheel is not controllable



2 CURVILINEAR APPROACH-BASED PROBLEM

Let considered a WAV with two controllable wheels and differential traction as shown in Fig. 1, whose configuration space $C = \mathbb{R}^2 \times S^1 \times T^2$ can be fully described by the generalized coordinates vector defined by $q = [x_1 \ x_2 \ \theta \ \phi_1 \ \phi_2]^T \in \mathbb{R}^5$ where $\{x_1, x_2, \theta\}$ describe the cartesian position and guidance of the body frame $\{L\}$ into the global and fixed frame $\{G\}$ while the set $\{\phi_1, \phi_2\}$ describe the angular position of each wheel. The wheels associated with angles ϕ_1, ϕ_2 are standard fixed wheels with five DOF, only rotation around the normal vector of the wheel plane and translation without steering on their individual frames, located at poses (x_{11}, x_{12}) and (x_{21}, x_{22}) with reference to the fixed global frame, respectively (both wheels frames have origin aligned with ground and origin of

the fixed global frame). None wheel can not be controlled independently. In front of the chassis, there is a castor wheel with free motion and uncontrollable (see Fig. 1). The kinematic constraints can be expressed like a pfaffian constraint $A^T(q) \dot{q} = 0$ where $A(q)$ is the matrix with the terms associated with non-integrable partial differential equations, generally associated with nonholonomic kinematic constraints of the contact's speed of the wheel into the frames $\{x11, x12\}$, $\{x21, x22\}$ with respect to $\{G\}$. Let this matrix be defined by

$$A(q) = \begin{bmatrix} \cos \theta & -\sin \theta & \sin \theta \\ \sin \theta & \cos \theta & -\cos \theta \\ 0 & b & b \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \triangleq \begin{bmatrix} A_1(\theta) \\ A_2(\theta) \end{bmatrix} \triangleq A(\theta),$$

where b is the displacement from each of driving wheels to the axis of symmetry of WAV and r is the radius of each wheel. Provided that the ideal kinematic constraints are not satisfied [i.e., $A^T(q) \dot{q} \neq 0$] then the generalized velocity vector \dot{q} may be written as

$$\dot{q} = S(q)v + A(q)\varepsilon\mu = \begin{bmatrix} -\sin \theta & 0 \\ \cos \theta & 0 \\ 0 & 1 \\ -1/r & -b/r \\ 1/r & -b/r \end{bmatrix} v + A(q)\varepsilon\mu = \begin{bmatrix} S_1(\theta) \\ S_2(\theta) \end{bmatrix} v + A(q)\varepsilon\mu \tag{1}$$

being $\mu = [\mu_1 \ \mu_2 \ \mu_3]^T$ an instrumental vector in sense of accessing the violations of the ideal kinematic constraints in the WAV [10, 31, 32], $v = [v_n \ \omega]^T$ is the vector that contains the linear (v_n) and angular (ω) velocities and $S(q)$ is the jacobian matrix. The term ε is a scale factor associated with the flexibility of the dynamic model [33, 34, 6]. On interesting theoretical point is that The matrices $A(q)$ and $S(q)$ satisfy $A^T(q)S(q) = 0$, due that each row of $A^T(q)$ is orthogonal with each column of $S(q)$ [35, 29]. As usual, the dynamic model for a WAV is given by

$$B(q)u_\varepsilon = M(q)\ddot{q} + c(q, \dot{q}) + A(q)\lambda \tag{2}$$

where $M(q)$ represents the inertia of WAV, $c(q, \dot{q}) \in \mathbb{R}^2$ contains the centripetal and coriolis torques (it is assumed that the geometrical center coincides with the mass center, thus the centripetal and coriolis accelerations are negligible), $B(q)$ is a full rank matrix, the vector u_ε represents the input torques provided by the actuators and $\lambda \in \mathbb{R}^3$ represents the Lagrange multipliers associated to constraint forces [5]. In practice, the ideal constraints $A^T(q) \dot{q} = 0$ does not hold. So, by multiplying

both sides of (1) by $A^T(q)$, and by using $A^T(q)S(q) = 0$ (since one-forms in $A(q)$ as well as one-forms in $S(q)$ are mutually annihilated) is obtained that

$$A^T(q)\dot{q} = A^T(q)A(q)\varepsilon\mu. \tag{3}$$

Assumption 1. Assume that the norm of $A^T(q)A(q)\varepsilon\mu$ is limited, i.e., $\|A^T(q)A(q)\varepsilon\mu\| \leq \xi$, where ξ is a non-negative known function which depends on the lateral acceleration of the robot and the deformation of the wheels.

If $\varepsilon = 0$ then (3) becomes the ideal constraints. In other words, the parameter ε governs the dissatisfaction of the kinematic constraints and it must be included into the dynamic model. To this end, we propose a singularly perturbed dynamic model for the WAV, like in [30, 6, 7], defined by the following state-space:

$$\begin{cases} \dot{x} = B_0(q)v + [\varepsilon B_1(q) + B_2(q)]\mu + B_3(q)u_c & (4) \\ \varepsilon\dot{\mu} = C_0(q)v + [\varepsilon C_1(q) + C_2(q)]\mu + C_3(q)u_c & (5) \\ y = P_0(q) & (6) \end{cases}$$

where $x = [q^T v^T]^T$ can be used to denote the “slow” variables and μ beyond its instrumental meaning can be used to denote the “fast” variables; $u_c = [u_{c,1} \ u_{c,2}]^T$ has the manipulated inputs associated with the torques at the motors and $y = [y_1 \ y_2]^T$ has the cartesian coordinates of a point p located at a distance l of the symmetry axis of the WAV, i.e., it was defined:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = P_0(q) \triangleq \begin{bmatrix} x_1 - l \sin \theta \\ x_2 + l \cos \theta \end{bmatrix}. \tag{7}$$

When $\varepsilon = 0$ the model defined by (4)-(6) is called rigid model. When $\varepsilon \neq 0$ the model is called flexible model [29]. The matrices $B_i(q)$, $C_i(q)$, for $i = 0, 1, 2, 3$, are successively:

$$B_0(q) = \begin{bmatrix} S(\theta) \\ \Delta_0 \end{bmatrix}, \quad B_1(q) = \begin{bmatrix} A(\theta) \\ \Delta_1 \end{bmatrix}, \quad B_2(q) = \begin{bmatrix} 0_{5 \times 3} \\ \Delta_2 \end{bmatrix}, \quad B_3(q) = \begin{bmatrix} 0_{5 \times 2} \\ \Delta_3 \end{bmatrix},$$

$$C_0(q) = \begin{bmatrix} -\dot{\theta} \cos 2\theta & 0 \\ 1/3 \dot{\theta} \sin 2\theta & 0 \\ -1/3 \dot{\theta} \sin 2\theta & 0 \end{bmatrix}, \quad C_1(q) = \begin{bmatrix} 0 & \dot{\theta} & -\dot{\theta} \\ 1/3 \dot{\theta} & 0 & 0 \\ 1/3 \dot{\theta} & 0 & 0 \end{bmatrix}, \quad C_2(q) = \begin{bmatrix} a_3 D_o & 0 & 0 \\ 0 & a_4 G_o & a_4 G_o \\ 0 & a_4 G_o & a_4 G_o \end{bmatrix}, \quad C_3(q) = \begin{bmatrix} 0 & 0 \\ a_1 & 0 \\ 0 & a_1 \end{bmatrix},$$

being

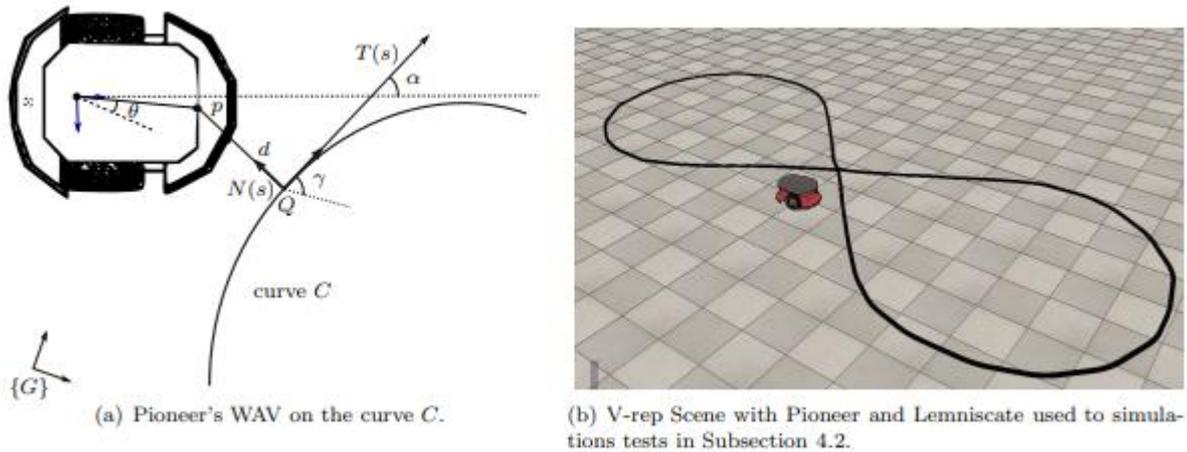
$$\Delta_0 = \begin{bmatrix} 1/3 \dot{\theta} \sin 2\theta & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} -1/3 \dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_2 G_o & a_2 G_o \end{bmatrix}, \quad \Delta_3 = \begin{bmatrix} -a_1 & a_1 \\ -a_1 & -a_1 \end{bmatrix}$$

with

$$a_1 = \frac{r^2}{3I_w}, \quad a_2 = -\frac{2I_w b^2 - 2I_c r^2}{I_c I_w (\delta + V)}, \quad a_3 = -\frac{4}{m (\delta + V)}, \quad a_4 = -\frac{2b^2}{I_c (\delta + V)} - \frac{r^2}{I_w (\delta + V)}$$

where the parameter V is the velocity of the wheel center and δ is a “small” positive constant to avoid the numerical problem for small values of V (i.e., for small values of V , it is replaced by $V + \delta$). The parameters

Figure 2: Configuration of the Pioneer’s WAV on the curve C by using a curve-viewpoint described by three variables, s, d, α .



D_o and G_o are normalized values defined by

$$D_o = \epsilon D \quad \text{and} \quad G_o = \epsilon G, \tag{8}$$

where D and G are the stiffness coefficients for the transversal and longitudinal movements of each wheel, respectively. Throughout the work reported here, it will be taken two assumptions: i) the longitudinal and transversal stiffness coefficients (G and D , respectively) are the same for the three wheels and $\epsilon = \inf\{1/G, 1/D\}$ and ii) the velocities of both driving wheels at their center are taken to be identical, and more precisely, equal to average velocity:

$$V = \left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{\theta}^2 \right)^{1/2}. \tag{9}$$

2.1 KINEMATIC CURVILINEAR MODEL ON TASK SPACE

The work reported here will use a scaling function $s : [0, T] \rightarrow [0, 1]$ on domain time for any known time T in order to achieve different configurations θ by using linear paths in the configuration space C . In this way, there is a function $q, q(s), q(s(t))$ for $\forall t \in [0, T]$. Normally s is associated with polynomial description on domain time (e.g., s-curve time scalings or third-order/fifth-order polynomials) but here it will be considered as a curvilinear coordinate along a differentiable simple curve C defined by one of its points, the unitary tangent vector at the point and its curvature $\kappa(s)$. The

following assumptions will be considered in order to make the controller design easy [9, 33, 34]. Without loss of generality, let $|\kappa(s)| < 1/R$, $\forall s$ where $R > 0$ is a constant. For a given point Q in the curve C , assume the curvilinear coordinate at Q is s , and $\{Q, T(s), N(s)\}$ is the Frenet frame on the curve at point Q , being $T(s)$ the tangent vector at point Q and $N(s)$ the normal vector at same point (see Fig. 2). The distance between point p and the curve C is smaller than R , thus the projection of point p on the curve is unique and denoted as Q . Now, let $|pQ|$ be the distance between the two points p and Q , α be the orientation of the WAV with respect to the tangent vector $T(s)$ of the curve C at point Q , given a desired velocity $v * n > 0$ (see Fig. 2). The control problem considered in this paper is finding a controller $u \in \mathbb{R}^3$ for system (4)-(6) such that $|pQ|$, $|\alpha|$ and $|v_n - v * n|$ are as small as possible when time approaches to the infinity. The position of point p is parameterized by (s, d) , where d is the coordinate of point p along $N(s)$. Noting $\alpha = \theta - \gamma$, the WAV's configuration can be now parameterized by $p = [p_1, p_2, p_3]^T$ $T = [s, d, \alpha]^T$.

Usually, by using classic mechanics and also proposed in [36, 9, 18, 37, 38]:

$$\dot{p}_1 = \frac{v_n \cos p_3 + \varepsilon \sin p_3}{1 - \kappa(p_1)p_2} \quad (10)$$

$$\dot{p}_2 = v_n \sin p_3 - \varepsilon \cos p_3 \quad (11)$$

$$\dot{p}_3 = \omega - \frac{v_n \kappa(p_1) \cos p_3}{1 - \kappa(p_1)p_2} - \frac{\varepsilon \kappa(p_1) \sin p_3}{1 - \kappa(p_1)p_2} \quad (12)$$

From now on, the speed v_n will be considered as a function on s which may depend on the variables in p . Noting that equations (10)-(12) are well-defined if $|p_2| < R$. In this way, the synthesized control law will guarantee such condition.

3 ORIENTED MANIFOLD: LIE GROUP APPROACH

The controller is designed in two parts. The first part, a kinematic controller for subsystem defined by (10)-(12) is designed with the aid of an appropriate transformation. In the second part, a robust nonlinear state feedback based controller is proposed with the aid of the inverse dynamics and the controller obtained in the first part. Taking into account the proposal in [32], let M an oriented manifold such that the set of the vector fields twice differentiable, $X_\infty(M)$, is a Lie algebra with infinity dimension. Any open set $U \subset M$ will be described by p_1, p_2 and p_3 . In this way, the following proposals are made about M :

1. Let $\Omega^k c(M)$ be the set of k -forms on M with compact support. For any $w \in \Omega^k c(M)$ the exterior derivative of w satisfies $dw \neq 0$ and

$$\int_M dw = \int_{\partial M} w,$$

being ∂M the boundary of M

2. On M , there are three functions $G_1(p), G_2(p), G_3(p) \in \Omega^0 c(M)$ such that

$$dG_i(p) = \sum_{j=1}^3 g_{ij}(p) dp^j$$

where, according to [31, 32], $g_{11} = 1, g_{12} = (1 - \kappa(p_1)p_2) \tan p_3, g_{13} = -\kappa(p_1), g_{21} = g_{22} = 0, g_{23} = 1, g_{31} = \sin p_3, g_{32} = -\cos p_3, g_{33} = -\kappa(p_1) \sin p_3, 1 - \kappa(p_1)p_2$.

3. For any vector field proposed above the Lie bracket $[\cdot, \cdot] : X^\infty(M) \times X^\infty(M) \rightarrow X^\infty(M)$ is well-defined and it can be used as Lie derivative, i.e, $LX(Y) = [X, Y]$, for $\forall X, Y \in X^\infty(M)$.
4. The vector fields $X_i(p) \in X^\infty(M) : M \supset U \rightarrow \mathbb{R}^3 ; i = 1, \dots, 3$ are continuously differentiable on parameters $(p_1, p_2, p_3) \in U = D_1 \times D_2 \times D_3$ being $D_1, D_2, D_3 \in \mathbb{R}$ open and convex sets. This vector fields are defined formally as

$$X_i(p) = [g_{i1} \ g_{i2} \ g_{i3}]^T.$$

5. Let $E(p) \in X^\infty(M) : M \supset U \rightarrow \mathbb{R}^3$ be the error of the trajectory tracking associated with p , formally defined as a vector field $E : C^\infty(M) \rightarrow C^\infty(M)$ in M such that it is continuously differentiable on parameters $(p_1, p_2, p_3) \in U$ and it can be used like differential operator that satisfies Leibniz rule, i.e, $E(f) = L(f)$, for $\forall f \in C^\infty(M)$ and $L(\cdot)$ denotes symbolically the Leibniz rule. This vector field is defined formally as

$$E(p) = \left[\begin{array}{c} p_1 \\ X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} + k_2 \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} + \frac{\xi}{v_n^*} \sec^2 \left(\frac{\pi p_2}{2R} \right) \tanh \left(\frac{2R\xi}{\pi \delta_1} \tan \frac{\pi p_2}{2R} \sec^2 \left(\frac{\pi p_2}{2R} \right) \right) \end{array} \right]$$

Taking into account the inequalities in [40], it can be stated that

$$\begin{aligned}
 & -\frac{2R\xi}{\pi} \tan \frac{\pi p_2}{2R} \sec^2 \left(\frac{\pi p_2}{2R} \right) \tanh \left(\frac{2R\xi}{\pi \delta_1} \tan \frac{\pi p_2}{2R} \sec^2 \left(\frac{\pi p_2}{2R} \right) \right) + \left| \frac{2R\xi}{\pi} \tan \frac{\pi p_2}{2R} \left[X_3, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right| \leq \rho \delta_1 \\
 & -E_3(p)\xi \left(\left[X_3, \left[X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right] + \phi \right) \tanh \left(\frac{E_3(p)\xi}{\delta_1} \left[X_3, \left[X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right] + \frac{E_3(p)\xi \phi}{\delta_1} \right) \\
 & \qquad \qquad \qquad + \left| E_3(p)\xi \left(\left[X_3, \left[X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right] + \phi \right) \right| \leq \rho \delta_1,
 \end{aligned}$$

where ρ is a constant which satisfies $\rho = e^{-(\rho+1)}$ (i.e., $\rho = 0.2785$). Thus

$$\dot{V}(p)|_{\mathcal{U}-D_3} \leq -k_2 v_n^* \left(\frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right)^2 - k_3 v_n^* E_3^2(p) + 2\rho \delta_1 \leq -2 \min\{k_2 \delta_v, k_3 \delta_v\} V(p)|_{\mathcal{U}-D_3} + 2\rho \delta_1,$$

i.e., $V(p)$ constrained to set $D1 \times D2$ converges exponentially to a small ball containing the origin with maximum convergence rate $\min\{2k_2 \delta_v, 2k_3 \delta_v\}$. So, by assuming $v * n > \delta_v > 0$ and if

$$\begin{aligned}
 W(p, v) = & \left[\begin{array}{l} v_n^* \\ -\xi \left(\left[X_3, \left[X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right] + \phi \right) \tanh \left(\frac{E_3(p)\xi}{\delta_1} \left[X_3, \left[X_1, \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} \right] \right] + \frac{E_3(p)\xi \phi}{\delta_1} \right) \\ \dots \\ -k_3 E_3(p) v_n^* - \frac{2R}{\pi} \tan \frac{\pi p_2}{2R} v_n^* + \frac{\dot{v}_n^* \xi}{(v_n^*)^2} \sec^2 \left(\frac{\pi p_2}{2R} \right) \tanh \left(\frac{2R\xi}{\pi \delta_1} \tan \frac{\pi p_2}{2R} \sec^2 \left(\frac{\pi p_2}{2R} \right) \right) \end{array} \right] \quad (15)
 \end{aligned}$$

In this way, the radius of the ball can be adjusted by the parameter δ_1 to guarantee that $E_2(p)$ and $E_3(p)$ converges. Now, from (13) and (15) is obtained that

$$v = \Pi_2(p)W(p, v) \tag{16}$$

The equation (16) is so-called the kinematic controller for the WAV.

3.2 DYNAMIC CONTROLLER

Like in [6, 29], assuming that the control input u_ϵ is a smooth function of time u_ϵ , $u_\epsilon(q, v)$ then, for $\epsilon = 0$, the equation (5) can be rewritten as follows:

$$C_0(q)v + C_2(q)\mu + C_3(q)u_\epsilon(q, v) = 0, \tag{17}$$

The model defined by (4)-(6) is in standard form if only if (17) has $k \geq 1$ distinct isolated roots. Indeed, the root of (17), here denoted by $\bar{\mu}$, is

$$\bar{\mu} = -C_2^{-1}(q) [C_3(q)u_\varepsilon(q, v) + C_0(q)v], \tag{18}$$

thus the reduced system associated is obtained by substituting (18) in (4):

$$\dot{\bar{x}} = B_0(q)v - [\varepsilon B_1(q) + B_2(q)] C_2^{-1}(q) [C_3(q)u_\varepsilon(q, v) + C_0(q)v] + B_3(q)u_\varepsilon; \text{ for } \bar{x}(0) = x_0, \tag{19}$$

and the boundary layer system is

$$\frac{d\hat{\mu}}{d\tau} = C_0(q)v_0 + [\varepsilon C_1(q_0) + C_2(q_0)] (\hat{\mu} + \bar{\mu}) + C_3(q_0)u_\varepsilon; \text{ for } \hat{\mu}(0) = \mu_0 - \bar{\mu}_0, \tag{20}$$

where $\tau = t/\varepsilon$, v_0, q_0 are interpreted as fixed parameters and $\hat{\mu} = \mu - \bar{\mu}$ being $\bar{\mu}_0$ equal to (18) evaluated in v_0, q_0 .

Now, it can be introduced some conditions that are widely treated in [6, 30]. Initially it can assume that there are $T, \lambda_1, \lambda_2, \varepsilon_0$ and the balls $Z_1 = (0; \lambda_1), Z_2 = (0; \lambda_2)$ such that i) the matrices $B_i(q)$ and $C_i(q)$ in the model (4)-(6) (for $i = 0, \dots, 3$) and their partial derivatives with respect to x, μ and ε are continuous in $Z_1 \times Z_2 \times [0, \varepsilon_0] \times [0, T]$, ii) the function (18) and $\varepsilon C_1(q) + C_2(q)$ have continuous first partial derivatives, iii) the reduced system (19) has an unique solution \bar{x} defined on $[0, T]$ which belongs to Z_1 , iv) $\hat{\mu} = 0$ is an exponentially stable equilibrium point of the boundary layer system (20) uniformly in the parameter x_0 . Furthermore, $\mu_0 - \bar{\mu}(0)$ belongs to its domain of attraction. This condition implies that $\hat{\mu}(\tau)$ exists for $\tau \geq 0$ and that $\lim_{\tau \rightarrow +\infty} \hat{\mu}(\tau) = 0$.

Complementarily, Tikhonov's theorem states the relation between x and \bar{x} on one hand and $\mu, \bar{\mu}$ and $\hat{\mu}$ on the other hand [6, 30, 31]. According of this theorem, for a system in a standard form, if the Conditions (i)-(iv) are satisfied, then there exist positive constants v_1, v_2 and ε^* such that if $\|x_0 - \bar{x}_0\| < v_1, \|\mu_0 - \bar{\mu}_0\| < v_2$ and $\varepsilon < \varepsilon^*$ then the following approximations are valid for $t \in [0, T]$:

$$x(t) = \bar{x}(t) + \mathcal{O}(\varepsilon) \tag{21}$$

$$\mu(t) = \bar{\mu}(t) + \hat{\mu}(\tau) + \mathcal{O}(\varepsilon) \tag{22}$$

where $\mathcal{O}(\varepsilon)$ represents a quantity of the order of ε . With the Tikhonov's theorem is guaranteed that there exists $t_1 > 0$ such that the approximation $\mu(t) = \bar{\mu}(t) + \mathcal{O}(\varepsilon)$ is valid for $t \in [t_1, T]$. Leaving only choose an appropriate value for ε .

The global feedback control $u_\varepsilon = u_\varepsilon(q, v)$ is projected by using the inverse dynamics of (2) and the second derivative of (1). Thus,

$$\ddot{q} = \left[\frac{\partial S}{\partial q} S(q)v \right] v + S(q)\dot{v}. \tag{23}$$

Eliminating Lagrange multipliers in (2) and using the relation (23) give

$$\dot{v} = [S^T(q)MS(q)]^{-1} S^T(q) \left[Bu_\varepsilon - M \left[\frac{\partial S}{\partial q} S(q)v \right] v \right], \tag{24}$$

consequently, the law u_ε is defined by the inverse of (24):

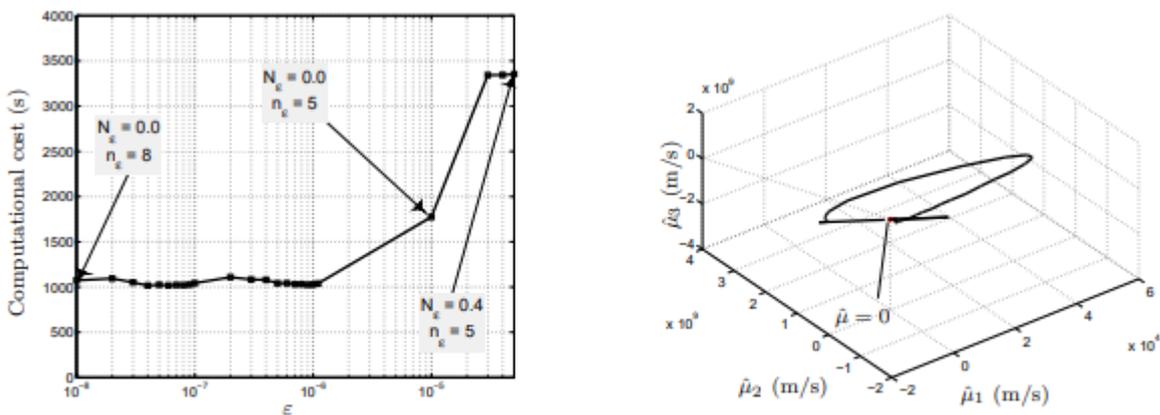
$$u_\varepsilon = [S^T(q)B(q)]^{-1} \left\{ S^T(q) \left[M(q)S(q)\rho M(q) \left[\frac{\partial S}{\partial q} S(q)v \right] v \right] \right\}. \tag{25}$$

By substituting (25) in (24) is obtained $\rho = \dot{v}$, and, by substituting (16) and (1) in this last equation yields,

$$\rho = \left\{ \frac{\partial \Pi(p)}{\partial p} \dot{p} \right\} W(p, v) + \Pi(p) \dot{W}(p, v) \tag{26}$$

where, according items (1)-(2), there are two functions $S1(p), S2(p) \in \Omega_0 c(M)$ such that there are two 1-forms $dSi(p) = h\nabla Si(p), dpi \in \Omega_1 c(M)$, for $\forall p \in M$ where $dp = [dp_1 dp_2 dp_3]^T$ represents a linear function on dual space $(\mathbb{R}^3)^*$, i.e, $dpi(p) = pi$. Thus, the jacobian operator $\partial \Pi(p)/\partial p$ can also be written as $[\nabla S1(p)^T \nabla S2(p)^T]^T$. The proposed controller in (25) and (26) combines existing results in singular

Figure 3: Simulation of the computational cost of the control law (25) according to the values of ε defined by $\varepsilon = 10^{-n_\varepsilon + N_\varepsilon 10^{-n_\varepsilon + 1}}$ and evolution of the boundary layer system.



(a) Simulation of the computational cost of the control law (25). Imminent instability when $\varepsilon \geq 5 \times 10^{-5}$ ($N_\varepsilon = 0.4$ and $n_\varepsilon = 5$).

(b) Evolution of the vector $\hat{\mu}$.

techniques when the slipping is included into the dynamic model [29, 20] whenever the kinematic controller is synthesized on an oriented manifold M . Unlike contributions which use manifolds of μ in order to linearize the dynamic model [17, 41, 14, 15], the work reported here includes the flexibility (represented by parameter ε) within the kinematic model (see the inclusion of ε and ξ in (10)-(12) and (15), respectively).

4 ACCESSING THE CONTROLLER: SIMULATION RESULTS

In the kinematic controller, the transformation $E_2(p) = 2R \pi \tan \pi p / 2R$ guarantees that $|p_2| < R$ if $E_2(p)$ is bounded. On the other side, if it is necessary that $|p_2| < d_0 < R$ during the application of the control action $W(p)$ (where d_0 is a positive constant that represents the initial displacement of the point p) then it can be set as $E_2(p) = 2d_0 \pi \tan \pi p / 2d_0$. In order to yields coherent results it should be noted that the initial value of p_2 should be less than d_0 , otherwise, an open-loop control can be first applied to the system such that $p_2 < d_0$ (pre-controlling stage).

In order to verify effectiveness of the proposed controller, simulations were done by using the kinematic control defined by (26) and the robust nonlinear state feedback based controller defined by (25). However, we must first know the appropriate value for ε due that Tikhonov's theorem imposes the limit ε^* .

Like as previous works of the author, ε 's value has a strong relation with flexibility and deformation of the wheels, i.e., it can be said that the value of ε increases with the increasing of the computational cost of the control law (25). Thus, if the friction coefficient increases the wheel deformation increases and it is a significant explanation for the dead-zone nonlinearity in actuators, showing that the computational effort is associated with an attempt of the control law (25) to overcome the dead zone [42, 6].

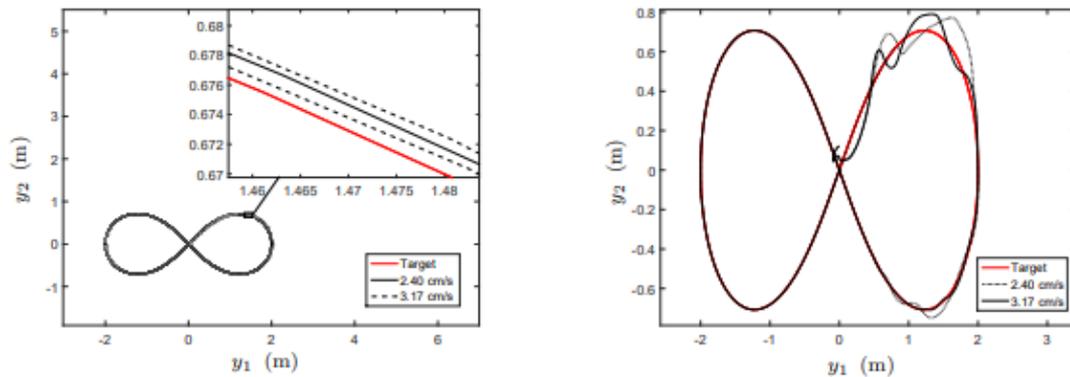
4.1. SETTING ε^*

Let considered the following transformation on ε for a better numerical manipulation:

$$\varepsilon = 10^{-n_\varepsilon} + N_\varepsilon 10^{-n_\varepsilon + 1} \quad (27)$$

being $n_\varepsilon \in \mathbb{Z}^+$ and $N_\varepsilon \in [0, 1] \subset \mathbb{R}^+$. Assuming that the coefficients D and G are the same for all wheels and by using of (8) it is chosen $D_0 = G_0 = 1$ N. It will be assume that the target trajectory is a lemniscate whose duration will be 4.5 s. The numerical values used in the simulations are as follows: $m = 17$ Kg,

Figure 4: Tracking trajectory for the WAV with a gradual increase in the velocity $v * n$.



(a) Tracking trajectory for the WAV with a gradual increase in the velocity v_n^* by using the control law (25) in the model (4)-(6) for $\epsilon \neq 0$.

(b) Tracking trajectory for the WAV with a gradual increase in the velocity v_n^* by using the control law (25) in the model (4)-(6) for $\epsilon = 0$.

$I_c = 0.537 \text{ Kg-m}^2$, $I_w = 0.0023 \text{ Kg-m}^2$ (by considering the WRM body like a solid cuboid shape with $0.24\text{m} \times 0.4\text{m} \times 0.45\text{m}$) $l = 0.2 \text{ m}$, $b = 0.24 \text{ m}$ and $r = 0.095 \text{ m}$. In the kinematic controller we choose $k_1 = k_2 = 1$, $\delta_1 = 0.01$ and $R = 105 \text{ m}$ due that the rhombus has four corners (i.e., $\kappa(s) \rightarrow \infty$), thus $R = 105 \text{ m}$ simulates a quasi-infinite curvature. To this end, it was used Simulink/Matlab in order to process data and plotting, however, V-REP platform was also used to verify behavior of Pioneer 3dX WAV according previous parameters.

In Fig. 3(a) is shown the computational cost (measured in seconds) for the interval $[10^{-8} , 5 \times 10^{-5}]$ in the domain of ϵ . It can be seen that the evolution of the computational cost increases when ϵ increases. According previous works it is well-known that for $\epsilon = 5 \times 10^{-5}$ ($N\epsilon = 0.4$ and $n\epsilon = 5$) the system becomes unstable [6] and the tracking is rather impracticable. Thus, it is defined $\epsilon^* = 5 \times 10^{-5}$ in order to remain coherent with Tikhonov’s theorem and such that any ϵ ’s value greater than ϵ^* will impose an unstable response to the system.

4.2 TRAJECTORY TRACKING

To observe the behavior of the control law (25) when it is applied in the model defined by (4)-(6), we can study the cases in which the model is totally rigid ($\epsilon = 0$) and flexible ($\epsilon \neq 0$), according to the Subsection 3.2. In Fig. 4(a) is shown the tracking made by the control law (25) when $\epsilon = 3 \times 10^{-5}$ such that the condition $\epsilon < \epsilon^*$ ($= 5 \times 10^{-5}$) is satisfied.

In Fig. 3(b) is shown the evolution of the vector $\hat{\mu}$ and it is proven that $\hat{\mu} = 0$ guarantee the condition (iv) at Subsection 3.2. Thus, it is possible to assert that the dynamic model defined by (4)-(6) satisfies the approximations (21)-(22) such that the slow manifold (related to μ) into the dynamic control is a suitable approach to linearizing the system on some local neighborhood. The Fig. 4(a) presents the tracking of the trajectory for two different velocities: 2.40 cm/s, 3.17 cm/s. It can be noted

that when the speed $v * n$ increases the deviations are also larger, i.e., the control law does not have enough robustness to follow the curves at certain parts of the lemniscate. When R is sufficiently great the contribution of slipping is significant. Then, a higher speed incurs a greater slipping. On another side, by setting $\varepsilon = 0$ the dynamic model becomes rigid. In Fig. 4(b) is shown the behavior of the tracking do not have a good performance, particularly at the transient response, with a greater deviation than the case $\varepsilon \neq 0$.

5 FINAL REMARKS

In the work reported here the trajectory tracking control problem of WAVs with slipping has been partially treated. To this end, a robust control law was composed of two approaches, one based on the wellknown singular perturbation approach by including the flexibility parameter into the dynamic model, see equations (4)-(6) (by using previous research of the author) and the other based on a curvilinear approach in order to synthesize the kinematic controller (see (24) and (25)). This last one had its foundation on the oriented-manifold M , which satisfies the properties on a Lie group (taking into account that the operations on M must be differentiable), especially when the tracking error and control actions (i.e., $E(p)$, $W(p)$) are assumed vector fields strictly associated with two 1-forms on an oriented-manifold M . The control law (25) was used in the dynamic model for the cases when $\varepsilon = 0$ (totally rigid) and when $\varepsilon \neq 0$ (flexible). The results observed in Subsection 4.2 indicates that the consideration of the flexible system is better than the rigid system.

REFERENCES

- [1] P. Morin, C. Samson, Control of Nonholonomic Mobile Robots Based on the Transverse Function Approach, *IEEE Transactions on Robotics* 25 (5) (2009) 1058–1073.
- [2] N. M. H. Norsahperi, K. A. Danapalasingam, A comparative study of LQR and integral sliding mode control strategies for position tracking control of robotic manipulators, *International journal of electrical and computer engineering systems* 10 (2) (2020) 73–83. doi:10.32985/ijeces.10.2.3.
- [3] M. Spong, On the force control problem for flexible joint manipulators, *IEEE Transactions on Automatic Control* 34 (1) (1989) 107–111.
- [4] R. R. Selmic, F. L. Lewis, Deadzone compensation in motion control systems using neural networks, *IEEE Transactions on Automatic Control* 45 (4) (2000) 602–613.
- [5] A. M. Bloch, J. Baillieul, P. Crouch, J. Marsden, *Nonholonomic Mechanics and Control*, Springer, New York, NY, USA, 2000.
- [6] C. P. Fernández, J. Cerqueira, A. Lima, Nonlinear trajectory tracking controller for wheeled mobile robots by using a flexible auxiliary law based on slipping and skidding variations, *Robotics and Autonomous Systems* 118 (2019) 231–250. doi:10.1016/j.robot.2019.05.007.
- [7] C. A. P. Fernández, A. B. B. F. Cunha, ROS architecture for nonlinear control in singularly perturbed wheeled mobile robots: Dynamic confined space-based approach, in: 2019 Latin American Robotics Symposium (LARS), 2019 Brazilian Symposium on Robotics (SBR) and 2019 Workshop on Robotics in Education (WRE), IEEE, 2019. doi:10.1109/lars-sbr-wre48964.2019.00037.
- [8] R. W. Brockett, Asymptotic Stability and Feedback Stabilization, in: R. S. M. R. W. Brockett, H. J. Sussmann (Eds.), *Differential Geometric Control Theory*, Birkhauser, Boston, 1983, pp. 181–191.
- [9] W. Dong, K.-D. Kuhnert, Robust adaptive control of nonholonomic mobile robot with parameter and nonparameter uncertainties, *IEEE Transactions on Robotics* 21 (2) (2005) 261–266.
- [10] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Suitable control laws to path tracking in omnidirectional wheeled mobile robots supported by the measuring of the rolling performance, in: XI Simpósio Brasileiro de Automação Inteligente - SBAI 2013, Fortaleza - Brasil, 2013.
- [11] M. Spong, K. Khorasani, P. Kokotovic, An integral manifold approach to the feedback control of flexible joint robots, *IEEE Journal of Robotics and Automation* 3 (4) (1987) 291–300.
- [12] M. W. Spong, M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, 1989.
- [13] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Hybrid and flexible auxiliary control law for an omnidirectional mobile base by using a singular perturbations approach in ros (robot operating system), in: XXI Congresso Brasileiro de Automação - CBA 2016, 2016.
- [14] C. A. P. Fernández, J. J. F. Cerqueira, Stability analysis for mobile robots with different time-scales based on unsupervised competitive neural networks, in: *IEEE Proceedings of the 2017 Joint Conference on Robotics: SBR-LARS Robotics Symposium and Robocontrol*, Curitiba - Brazil, 2017. doi:10.1109/sbr-lars-r.2017.8215270.

- [15] C. A. P. Fern´andez, NARMA-L2-based control of wheeled mobile robots singularly perturbed by using neural networks: curvilinear approach, in: IEEE Proceedings of the 2017 Joint Conference on Robotics: SBR-LARS Robotics Symposium and Robocontrol, Curitiba - Brazil, 2017.
- [16] B. D’Andr´ea-Novel, G. Campion, G. Bastin, Control of nonholonomic wheeled mobile robots by state feedback linearization, *The International Journal of Robotics Research* 14 (6) (1995) 543–559.
- [17] I. Motte, G. Campion, A slow manifold approach for the control of mobile robots not satisfying the kinematic constraints, *IEEE Transactions on Robotics and Automation* 16 (6) (2000) 875–880.
- [18] W. Dong, Y. Guo, Dynamic tracking control of uncertain nonholonomic mobile robots, in: 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2005, pp. 2774–2779.
- [19] W. Dong, Control of uncertain wheeled mobile robots with slipping, in: 49th IEEE Conference on Decision and Control, 2010, pp. 7190–7195.
- [20] W. Leroquais, B. D’Andrea-Novel, Modeling and control of wheeled mobile robots not satisfying ideal velocity constraints: the unicycle case, in: *Conf. Rec. IEEE/CDC*, Vol. 2, 1996, pp. 1437–1442.
- [21] C. A. P. Fern´andez, J. J. F. Cerqueira, A. M. N. Lima, Dinˆamica n˜ao-linear do escorregamento de um robˆo m´ovel omnidirecional com restri,c˜ao de rolamento, in: XIX Congresso Brasileiro de Autom´atica - CBA 2012, Campina Grande - Brasil, 2012.
- [22] Z.-P. Jiang, Lyapunov design of global state and output feedback trackers for non-holonomic control systems, *International Journal of Control* 73 (9) (2000) 744–761.
- [23] Z.-P. Jiang, H. Nijmeijer, A recursive technique for tracking control of nonholonomic systems in chained form, *IEEE Transactions on Automatic Control* 44 (2) (1999) 265–279.
- [24] I. Kolmanovsky, N. McClamroch, Developments in nonholonomic control problems, *IEEE Control Systems* 15 (6) (1995) 20–36.
- [25] G. Walsh, D. Tilbury, S. Sastry, R. Murray, J.-P. Laumond, Stabilization of trajectories for systems with nonholonomic constraints, *IEEE Transactions on Automatic Control* 39 (1) (1994) 216–222.
- [26] H. Yang, S. Wang, Z. Zuo, P. Li, Trajectory tracking for a wheeled mobile robot with an omnidirectional wheel on uneven ground 14 (7) (2020) 921–929. doi:10.1049/iet-cta.2019.1074.
- [27] L. Zhao, J. Jin, J. Gong, Robust zeroing neural network for fixed-time kinematic control of wheeled mobile robot in noise-polluted environment 185 (2021) 289–307. doi:10.1016/j.matcom.2020.12.030.
- [28] X. Gao, L. Yan, C. Gerada, Modeling and analysis in trajectory tracking control for wheeled mobile robots with wheel skidding and slipping: Disturbance rejection perspective 10 (9) (2021) 222. doi:10.3390/act10090222.
- [29] B. D’Andr´ea-Novel, G. Campion, G. Bastin, Control of wheeled mobile robots not satisfying ideal velocity constraints: A singular perturbation approach, *International Journal of Robust and Nonlinear Control* 5 (4) (1995) 243–267.
- [30] C. A. P. Fern´andez, J. J. F. Cerqueira, A. M. N. Lima, Robotics, Springer-Verlag, 2015, Ch. Trajectory control of wheeled mobile robots not satisfying ideal velocity constraints by using slipping and skidding variations: a singular perturbation approach, pp. 74–95.

- [31] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Tracking trajectory control of wheeled mobile robots singularly perturbed by using slipping and skidding variations: curvilinear coordinates approach (part ii), in: Proceedings of 11th IFAC Symposium on Robot Control, 2015.
- [32] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Tracking trajectory control of wheeled mobile robots singularly perturbed by using slipping and skidding variations: curvilinear coordinates approach (part i), in: Proceedings of 11th IFAC Symposium on Robot Control, 2015.
- [33] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Trajectory tracking control of an omnidirectional wheeled mobile robot with slip and deformation: a singular perturbation approach, in: XX Congresso Brasileiro de Automática - CBA 2014, 2014.
- [34] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Trajectory tracking control of nonholonomic wheeled mobile robots with slipping on curvilinear coordinates: a singular perturbation approach, in: XX Congresso Brasileiro de Automática - CBA 2014, 2014.
- [35] R. M. Murray, Z. Li, S. S. Sastry, A Mathematical Introduction to Robotic Manipulation, First Edition, CRC Press LLC, 1994.
- [36] W. Dong, W. Huo, S. Tso, W. Xu, Tracking control of uncertain dynamic nonholonomic system and its application to wheeled mobile robots, IEEE Transactions on Robotics and Automation 16 (6) (2000) 870–874. doi:10.1109/70.897799.
- [37] C. Samson, K. Ait-Abderrahim, Mobile robot control. part 1: Feedback control of nonholonomic wheeled cart in cartesian space, INRIA Report 1288 (1990).
- [38] C. Samson, Control of chained systems application to path following and time-varying point-stabilization of mobile robots, IEEE Transactions on Automatic Control 40 (1) (1995) 64–77. doi:10.1109/9.362899.
- [39] C. A. P. Fernández, J. J. F. Cerqueira, A. M. N. Lima, Regulable convergence radius for flexible auxiliary control law in wheeled mobile robots singularly perturbed: curvilinear approach, in: XII Simpósio Brasileiro de Automação Inteligente, Natal - Brasil., 2015.
- [40] M. M. Polycarpou, Stable adaptive neural control scheme for nonlinear systems 41 (3) (1996) 447–451. doi:10.1109/9.486648.
- [41] C. A. P. Fernández, Stabilizing model predictive control for wheeled mobile robots with linear parameter-varying and different time-scales, in: 2018 Latin American Robotic Symposium, 2018 Brazilian Symposium on Robotics (SBR) and 2018 Workshop on Robotics in Education (WRE), IEEE, 2018. doi:10.1109/lars/sbr/wre.2018.00064.
- [42] C. A. P. Fernández, J. J. F. Cerqueira, Control de velocidad con compensación de deslizamiento en las ruedas de una base holonmica usando un neurocontrolador basado en el modelo narma-l2, in: IX Congresso Brasileiro de Redes Neurais, Ouro Preto - Brasil, 2009.